International Journal of Applied Mathematics & Statistical Sciences (IJAMSS)
ISSN(P): 2319-3972; ISSN(E): 2319-3980
Vol. 6, Issue 5, Aug – Sep 2017; 47-50

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

PFC-MODULE

BUTHAI NAHNEJAD SHIHAB

Department of Mathematics, College of Education, University of Baghdad, Iraq

ABSTRACT

The main objective of this paper is to introduce another type of fully cancellation (denoted it F-Cancellation) module, namely prime –Fully Cancellation (denoted as PFC-Module), and we study the relationships btween these two concepts. We investigate some results of such modules.

KEYWORDS: Fully Cancellation Module, Max-Fully Cancellation Module, Artinian Ring, Boolean Ring and PFC-Module

1. INTRODUCTION

Throughout this paper, all rings are commutative with identity and all modules are unitary. Let M be an R-module. Then, M is said to be fully cancellation module,. f for each ideal I of R and for each submodules N_1, N_2 of M such that, $IN_1=IN_2$ implies $N_1=N_2$ [1]. In this case, if for every non-zero maximal ideal I of R and for every submodules $N_1$and N_2 of M such that $IN_1=IN_2$, then $N_1=N_2$ and we call it maximal –fully cancellation module (denoted it by the symbol MFC-Modul [2]. Now in this paper, we define the concept of prime –Fully cancellation (denote it by the symbol PFC-Module), we give some equivalent conditions for a PFC-Modul.

Also, we will find some relations between max-fully cancellation module and PFC-Module.

2. MAIN RESULTS

Definition (2.1)

Let M be a R-module. M is called PFC-Module for every non zero prime ideal I of R and for every submodules N_1, N_2 of M such that $IN_1=IN_2$, then $N_1=N_2$.

Remarks and Examples (2.2)

(1) Z as the Z - module is a PFC - Module.

(2) Z_6 as a Z_6-module is not PFC-Module.Since $(\overline{3})$ is prime ideal of Z_6 and $(\overline{3})$, Z_6 are submodules of Z_6 such that $(\overline{3})$ $(\overline{3})=(\overline{3})Z_6$ Z s not prime-fully motion by the symbol MFC-Modul but $(\overline{3})\neq Z_6$.

(3) Every full cancellation module is a PFC - Module, but the converse is not true in general for example:-

Let R $=Z_{24}$ and M=$(\overline{3})$ as an R-module ,since $(\overline{2})$ is prime ideal of Z_{24} and $(\overline{9})$,$(\overline{21})$ are two submodules of $(\overline{3})$Such that $(\overline{2})(\overline{9})$= $(\overline{2})(\overline{21})$=$(\overline{18})$ .Then $(\overline{9})$ = $(\overline{21})$ .Whil it is not fully cancellation R-module. Since $(\overline{8})$ is a nonzero ideal of Z_{24} and $(\overline{3})$,$(\overline{0})$ are two submodules of $(\overline{3})$ Such that $(\overline{8})(\overline{3})$= $(\overline{8})(\overline{0})$= $(\overline{0})$,but $(\overline{3})\neq(\overline{0})$.

(4) Every submodule of a PFC-Module is a PFC - Module.

(5) Let $M_1$and M_2 be an R-modules such that M_1 (M_2.Then M_1 is a PFC - module if and only if M_2 is

PFC-Module.

The Following Theorem is a Characterization of PFC-Module:

Theorem (2.3)

Let M be an R-module, let N_1, $N_2$are two submodules of M, let I be a non zero prime ideal of R, then the following statements are equivalent:-

(1) M is an MFC - Module.

(2) if $IN_1 \subseteq IN_2$, then $N_1 \subseteq N_2$.

3) if $I\prec a\succ \subseteq IN_2$, then $a \in N_2$where $a \in M$.

(4) $(IN_1{:}R\ IN_2) = (N_1{:}_R\ N_2)$

Proof

1) $\Rightarrow$ (2) If $IN_1 \subseteq IN_2$ then $IN_2 = IN_1 + IN_2$ Which Implies $IN_2 = I(N_1 + N_2)$,

But M is PFC-Module, then $N_2 = (N_1 + N_2)$ and hence $N_1 \subseteq N_2$

If $I\prec a\succ \subseteq IN_2$ then $\prec a\succ \subseteq N_2$ by (2) Which implies, $a \in N_2$. (2)$\Rightarrow$ (3)

(3) $\Rightarrow$ (4) If $IN_1 = IN_2$, To prove that $N_1 = N_2$. Let $a \in N_1$ then $I\prec a\succ \subseteq IN1 \subseteq IN_2$And hence $a \in N_2$ by (3) Similarly , we can show $N2 \subseteq N_1$. Thus $N_1 = N_2$.

(1) $\Rightarrow$ (4) Let $r \in (IN_1{:}R\ IN_2)$,.Then $r\ IN_2 \subseteq IN_1$ So, $IrN_2 \subseteq IN_1$ and since (1) implies (2), we have $N_2 \subseteq N_1$.

Thus $r \in (N_1{:}\_RN_2)$ and hence $(IN_1{:}R\ IN_2) \subseteq (N_1{:}\ RN_2)$

Let $r \in (N_1{:}\ _RN_2)$. Then $rN_2 \subseteq N_1$ which implies $IrN_2 \subseteq IN_1$ and hence $rIN_2 \subseteq IN_1$.

Therefore $r \in (IN_1{:}RIN_2)$ and hence $(N1{:}RN2) \subseteq (IN_1{:}RIN_2)$. Then we get $(N_1{:}\ RN_2) = (IN_1{:}RIN_2)$

(4)$\Rightarrow$(1)

Let $IN_1 = IN_2$ Then by (4) $(IN_1{:}R\ IN_2) = (N_1{:}RN_2)$. But $(IN_1{:}R\ IN_2) = R$

(Since $IN_1 = IN_2$). Then $(N_1{:}R\ N_2) = R$ so $N_2 \subseteq N_1$. Similarly $(IN_2{:}R\ IN_1) = (N_2{:}\ RN_1)$

Thus $(N_2{:}R\ N_1) = R$ Which implies $N1 \subseteq N_2$. Therefore $N_1 = N_2$..

Before we give our proposition, the following concepts are needed.

A ring R is called a Boolean ring, in case, each of its elements is an idempotent. And, a commutative ring R with unity is called an Artinian ring, if and only if for any descending chain of ideals $I1 \supseteq I2 \supseteq I3 \supseteq \ldots\ldots..$ of R $\exists n \in Z+$ such that $In = In+1 = \ldots\ldots.$ [3]

Now, the following proposition gives the relationship betweenMFC-Module and PFC-Module.

Proposition (2.4)

Every PFC-Module is max-fully cancellation module

Proof

It is easy

The converse of proposition (2.4) is true under the condition that the ring R is PID or regular or Artinian or Boolean ring.

Proposition (2.5)

Let R be a PID (regular or Artinian or Boolean) and M be an R-module.

Then, is PFC-Module if and only if M is MFC-Module.

Proof

It is obvious

Proposition (2.6)

Let M be a MFC-Module over a ring . If M is a cancellation module, then every non zero maximal ideal of R is cancellation ideal.

Proof

Let I be a nonzero maximal ideal of R, such that AI=BI, where A, B is two ideals of R. Now, we have AIM=BIM, then IAM=IBM. But M is an MFC - Module,

Therefore, AM=BM. As M is cancellation module, then A=B by [4].

Proposition (2.7)

Let M, N be two R-modules. If M≅N, then M is PFC-Module if

and only if N is a PFC - Module.

Proof

Let θ: M⟶N be an isomorphism. Suppose M is a MFC-Module

To prove N is a MFC-Module,

For every non zero prime ideal I of R and every submodules N_1, N_2 of N.Let $I\overline{N_1}=I\overline{N_2}$

Now, there exists two submodules N_1, N_2 of M such that θ (N1) = $\overline{N_1}$, θ(N_2)= $\overline{N_2}$

Then I θ(N_1) = I θ(N_2) ,, Which implies θ(I N_1)= θ(I N_2). Therefore IN^1 = IN_2

since θ is (1-1))But M is PFC-Module. Then N_1=N_2 and hence

θ(N_1)= θ(N_2) Therefore $\overline{N_1}$ = $\overline{N_2}$ That is N is PFC-Module.

Conversely

Suppose that N is PFC-Module over the a ring. Let IN_1 = IN_2 for every non Zero prime ideal I of R and every submodules N_1, N_2 of M. Now, θ(I N_1)= θ(I N_2). Which implies I θ(N_1)= I θ(N_2), where θ(N_1), θ(N_2) are two submodules of N

Also N is a PFC - Module. Then $\theta(N_1)= \theta(N_2)$ Which implies $N_1=N_2$

since θ is (1-1)) Which completes the proof.

CONCLUSIONS

This study was conducted to introduce a different type of fully cancellation module, denoted as F-Cancellation, namely, prime–Fully Cancellation (denoted as PFC-Module). The relationship between the two concepts were investigated on these modules, and the results have been presnetd in this paper.

REFERENCES

1. I. M. A. Hadi and A.A. Elewi, (2014)"Fully cancellation and Naturaly cancellation modules" Journal of Al-Nahrain University, 17, (3), September (2014), pp. 178-184.
2. Bothaynah.N,. Shihab and Heba.M.A., Judi (2015)" Max-Fully cancellation modules" Journal of Advances Mathematics Voi (11).No. (7).PP 5462-5475.
3. F. W.. Anderson and K.R., Fuller. (1973)"Rings and Categories of Modules" University of Oregon,
4. Bothaynah N. Shihab (2000)" On Restricted Cancellation module", M.Sc. Thesis, University of Tikret.